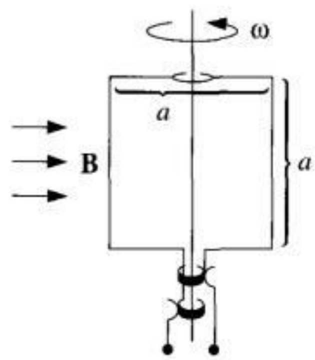

Figure 7.18

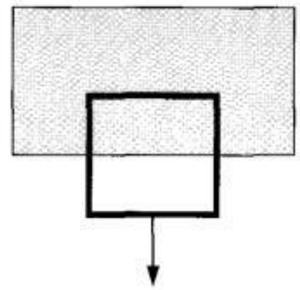

Figure 7.19

7.2 Electromagnetic Induction

7.2.1 Faraday's Law

In 1831 Michael Faraday reported on a series of experiments, including three that (with some violence to history) can be characterized as follows:

Experiment 1. He pulled a loop of wire to the right through a magnetic field (Fig. 7.20a). A current flowed in the loop.

Experiment 2. He moved the *magnet* to the *left*, holding the loop still (Fig. 7.20b). Again, a current flowed in the loop.

Experiment 3. With both the loop and the magnet at rest (Fig. 7.20c), he changed the *strength* of the field (he used an electromagnet, and varied the current in the coil). Once again, current flowed in the loop.

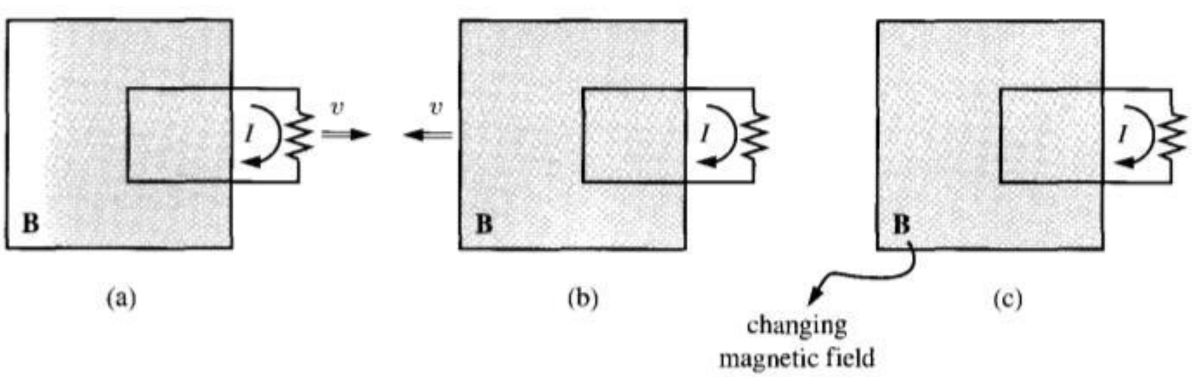

Figure 7.20

The first experiment, of course, is an example of motional emf, conveniently expressed by the flux rule:

$$\mathcal{E} = -\frac{d\Phi}{dt}.$$

I don't think it will surprise you to learn that exactly the same emf arises in Experiment 2—all that really matters is the *relative* motion of the magnet and the loop. Indeed, in the light of special relativity is *has* to be so. But Faraday knew nothing of relativity, and in classical electrodynamics this simple reciprocity is a coincidence, with remarkable implications. For if the *loop* moves, it's a *magnetic* force that sets up the emf, but if the loop is *stationary*, the force *cannot* be magnetic—stationary charges experience no magnetic forces. In that case, what *is* responsible? What sort of field exerts a force on charges at rest? Well, *electric* fields do, of course, but in this case there doesn't seem to be any electric field in sight.

Faraday had an ingenious inspiration:

A changing magnetic field induces an electric field.

It is this "induced" electric field that accounts for the emf in Experiment 2.[6] Indeed, if (as Faraday found empirically) the emf is again equal to the rate of change of the flux,

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}, \tag{7.14}$$

then $\mathbf{E}$ is related to the change in $\mathbf{B}$ by the equation

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}. \tag{7.15}$$

This is **Faraday's law**, in integral form. We can convert it to differential form by applying Stokes' theorem:

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.} \tag{7.16}$$

Note that Faraday's law reduces to the old rule $\oint \mathbf{E} \cdot d\mathbf{l} = 0$ (or, in differential form, $\nabla \times \mathbf{E} = 0$) in the static case (constant $\mathbf{B}$) as, of course, it should.

In Experiment 3 the magnetic field changes for entirely different reasons, but according to Faraday's law an electric field will again be induced, giving rise to an emf $-d\Phi/dt$. Indeed, one can subsume all three cases (and for that matter any combination of them) into a kind of **universal flux rule**:

> Whenever (and for whatever reason) the magnetic flux through a loop changes, an emf
>
> $$\mathcal{E} = -\frac{d\Phi}{dt} \tag{7.17}$$
>
> will appear in the loop.

[6]You might argue that the magnetic field in Experiment 2 is not really *changing*—just *moving*. What I mean is that if you sit at a *fixed location*, the field *does* change, as the magnet passes by.

Many people call *this* "Faraday's law." Maybe I'm overly fastidious, but I find this confusing. There are really *two* totally different mechanisms underlying Eq. 7.17, and to identify them both as "Faraday's law" is a little like saying that because identical twins look alike we ought to call them by the same name. In Faraday's first experiment it's the Lorentz force law at work; the emf is *magnetic*. But in the other two it's an *electric* field (induced by the changing magnetic field) that does the job. Viewed in this light, it is quite astonishing that all three processes yield the same formula for the emf. In fact, it was precisely this "coincidence" that led Einstein to the special theory of relativity—he sought a deeper understanding of what is, in classical electrodynamics, a peculiar accident. But that's a story for Chapter 12. In the meantime I shall reserve the term "Faraday's law" for electric fields induced by changing magnetic fields, and I do *not* regard Experiment 1 as an instance of Faraday's law.

Example 7.5

A long cylindrical magnet of length L and radius a carries a uniform magnetization $\mathbf{M}$ parallel to its axis. It passes at constant velocity v through a circular wire ring of slightly larger diameter (Fig. 7.21). Graph the emf induced in the ring, as a function of time.

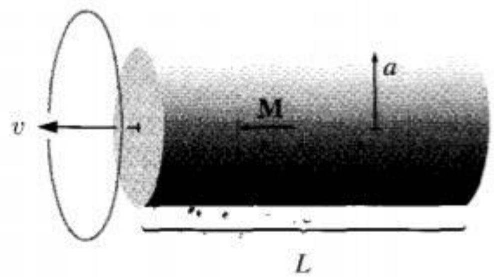

Figure 7.21

Solution: The magnetic field is the same as that of a long solenoid with surface current $\mathbf{K}_b = M\,\hat{\boldsymbol{\phi}}$. So the field inside is $\mathbf{B} = \mu_0\mathbf{M}$, except near the ends, where it starts to spread out. The flux through the ring is zero when the magnet is far away; it builds up to a maximum of $\mu_0 M\pi a^2$ as the leading end passes through; and it drops back to zero as the trailing end emerges (Fig. 7.22a). The emf is (minus) the derivative of Φ with respect to time, so it consists of two spikes, as shown in Fig. 7.22b.

Keeping track of the *signs* in Faraday's law can be a real headache. For instance, in Ex. 7.5 we would like to know which *way* around the ring the induced current flows. In principle, the right-hand rule does the job (we called Φ positive to the left, in Fig. 7.22a, so the positive direction for current in the ring is counterclockwise, as viewed from the left; since the first spike in Fig. 7.22b is *negative*, the first current pulse flows *clockwise*, and the second counterclockwise). But there's a handy rule, called **Lenz's law**, whose sole purpose is to help you get the directions right:[7]

[7] Lenz's law applies to *motional* emf's, too, but for them it is usually easier to get the direction of the current from the Lorentz force law.

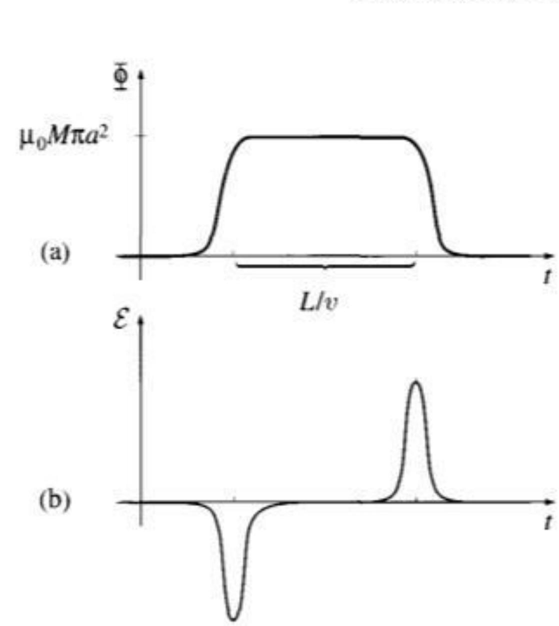

Figure 7.22

Nature abhors a change in flux.

The induced current will flow in such a direction that the flux *it* produces tends to cancel the change. (As the front end of the magnet in Ex. 7.5 enters the ring, the flux increases, so the current in the ring must generate a field to the *right*—it therefore flows *clockwise*.) Notice that it is the *change* in flux, not the flux itself, that nature abhors (when the tail end of the magnet exits the ring, the flux *drops*, so the induced current flows *counterclockwise*, in an effort to restore it). Faraday induction is a kind of "inertial" phenomenon: A conducting loop "likes" to maintain a constant flux through it; if you try to *change* the flux, the loop responds by sending a current around in such a direction as to frustrate your efforts. (It doesn't *succeed* completely; the flux produced by the induced current is typically only a tiny fraction of the original. All Lenz's law tells you is the *direction* of the flow.)

Example 7.6

The "jumping ring" demonstration. If you wind a solenoidal coil around an iron core (the iron is there to beef up the magnetic field), place a metal ring on top, and plug it in, the ring will jump several feet in the air (Fig. 7.23). Why?

Solution: *Before* you turned on the current, the flux through the ring was *zero*. *Afterward* a flux appeared (upward, in the diagram), and the emf generated in the ring led to a current (in the ring) which, according to Lenz's law, was in such a direction that *its* field tended to cancel this new flux. This means that the current in the loop is *opposite* to the current in the solenoid. And opposite currents repel, so the ring flies off.[8]
